$$H = \cos(\pi x'/a)$$
$$I = \sin(\pi y/b)$$
$$L = \cos(\pi y/b)$$
$$M = \sin(\pi y'/b)$$
$$N = \cos(\pi y'/b)$$
$$R_{mn} = \left[(x - x_m)^2 + (y - y_n)^2\right]^{1/2}$$
$$x_m = (m + 1/2)a + (-1)^m (x' - a/2)$$
$$y_m = (m + 1/2)b + (-1)^m (y' - b/2)$$
$$X_m = \pi(x - x_m)/b$$
$$Y_m = \pi(y - y_m)/a$$
$$S_m = \sinh(X_m)$$
$$T_m = \cosh(X_m)$$
$$U_m = \sinh(Y_m)$$
$$V_m = \cosh(Y_m).$$

Singularities in g and $\overline{G}_{st}$ are exhibited by the terms corresponding to $m = 0, n = 0$.

Appendix II

A. Semipositive Definiteness of Matrix C

Let us consider the quadratic form $x_t C x$ where x is any real N-dimensional vector different from zero. Due to (12b) we have

$$x_t C x = \sum_{i,j} x_i x_j C_{ij} = \int_\sigma \psi(l)\rho(l)\, dl \qquad \text{(A1)}$$

where

$$\rho(l) = \sum_i x_i \frac{\partial w_i}{\partial l}$$

$$\psi(l) = \int_\sigma g(s, s')\rho(l')\, dl'.$$

Due to the meaning of g, $\psi(l)$ is coincident with the electrostatic potential on σ given by a charge density ρ distributed on σ itself. Then (A1) is coincident with the expression of the electrostatic energy of this charge and, therefore, it is nonnegative. Null values of $x_t C x$ may occur only with a vector x such that $\rho(l)$ is zero, or, equivalently, such that $\sum x_i w_i =$ constant. Due to the choice of the basis $\{w_i\}$ such a vector exists if σ connects two points lying on the boundary of S_0 (see Section III). More generally, denoting by P the sum of the number of the portions of σ which connect points on the boundary plus the number of loops, P independent such vectors exist. Then C has P null eigenvalues and its rank is $R = N - P$.

B. Positive Definiteness of Matrix L′

It is easily deduced that a quadratic form assoicated with L' represents the electrostatic energy due to a charge density given by

$$\rho'(l) = \sum_i x_i u_i.$$

This cannot vanish, due to the independence of the functions u_i. Then, the quadratic form is always positive and matrix L' is positive-definite.

C. Positive Definiteness of Matrix L

Due to (12c), a generic quadratic form associated with L is

$$x_t L x = \sum_{i,j} \int_\sigma \int_\sigma u_i t(l) \cdot \overline{G}_{st}(s, s') \cdot t(l') u_j \, dl\, dl'. \qquad \text{(A2)}$$

The solenoidal dyad $\overline{G}_{st}$ may be expanded as

$$\overline{G}_{st}(r, r') = \sum_m \frac{e_m(r) e_m(r')}{k_m^2}. \qquad \text{(A3)}$$

This expansion is easily established starting from (5), taking into account that $\nabla \times \nabla \times e_m = k_m^2 e_m$, $n \times e_m = 0$ at the boundary, and using the property of eigenvectors e_m of being mutually orthogonal and orthogonal to $\nabla \nabla' g$. By substituting (A3) into (A2), we obtain

$$x_t L x = \sum_m \left(\int_\sigma f(l) \frac{t(l) \cdot e_m(s)}{k_m} dl \right)^2$$

where $f(l) = \sum_1^N x_i w_i(l)$. Since functions w_i are linearly independent $f(l)$ cannot vanish, so that the quadratic form is always positive. Then matrix L is positive-definite.

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Correction to "Quasi-Optical Method for Measuring the Complex Permittivity of Materials"

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In the above paper,[1] in Column 1 of Table II on page 663, Reference [9] should read [10] and Reference [10] should read [12].

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